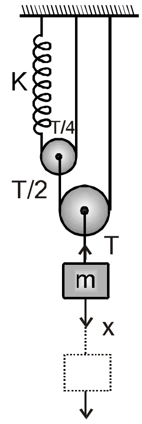
Solution of DPP # 6

TARGET : JEE (ADVANCED) 2015
COURSE : VIJAY & VIJETA (ADR & ADP)

PHYSICS

1. Let mass 'm' falls down by x so spring extends by 4x ;



$\therefore \ \frac{T}{4} = k(4x)$

T = (16k) x

Where T is the restoring force on mass m

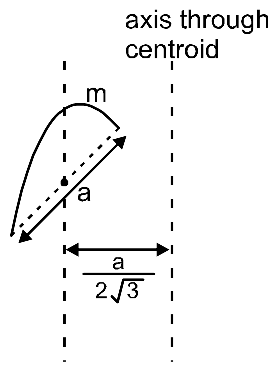
$\therefore \ f = \frac{1}{2\pi}\sqrt{\frac{16k}{m}}$

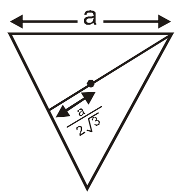
$f = \frac{2}{\pi}\sqrt{\frac{k}{m}} = \frac{2}{\pi} \times \sqrt{\frac{25}{1}} = \pi$ Hz

2. Apply C.O.A.M.,

$10 \times 1 = \frac{ML^2}{3}\omega$; ω = 15 rad. K.E. = $\frac{1}{2}I\omega^2$ = 75 J

3.

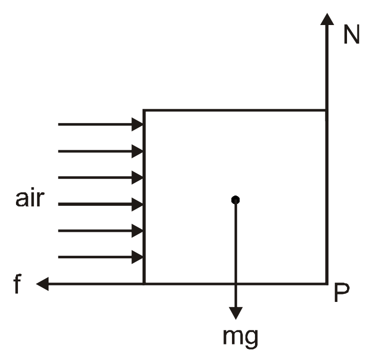




$I' = I_{cm} + m\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{5ma^2}{24}$, $\quad I_{cm} = \frac{m\left(\frac{a}{2}\right)^2}{2} = \frac{ma^2}{8}$

$I = 3I' = \frac{5ma^2}{8}$.

4. $F = V\frac{dm}{dt} = \rho_a \ell hV.V = \rho_a \ell hV^2$ $\qquad \{\because \frac{dm}{dt} = \rho_a \ell hV\}$



Total torque of air about point P is $\rho_a \ell hV^2 \frac{h}{2}$

$\tau_a = \frac{\rho_a \ell h^2 V^2}{2}$; $\tau_w = Mg.\frac{t}{2} = \rho_\omega.\ell.h.t.g.\frac{t}{2}$

for toppling $\quad \tau_a > \tau_\omega \quad \Rightarrow \quad V > \left(\frac{\rho_\omega g}{\rho_A h}\right)^{1/2}.t$

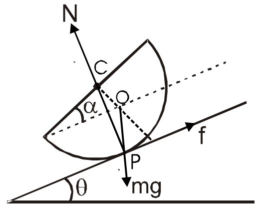
5. $x = A_0(1 + \cos 2\pi\upsilon_2 t).\ \sin 2\pi\upsilon_1 t$

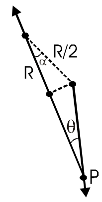
$= A_0 \sin 2\pi\upsilon_1 t + \frac{A_0}{2}[(\sin 2\pi\,(\upsilon_1 + \upsilon_2)t + \sin 2\pi\,(\upsilon_1 - \upsilon_2)t]$

Hence the frequencies are

$\upsilon_1,\ |\upsilon_1 - \upsilon_2|,\ \upsilon_1 + \upsilon_2$.

6. O is the centre of mass of the hollow hemisphere and is $\frac{R}{2}$ from C.





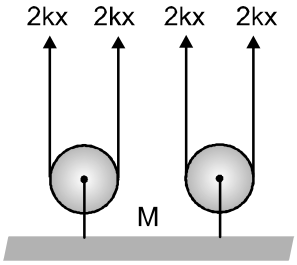
$f = mg \sin\theta$ .... (1)

$N = mg \cos\theta$ .... (2)

$N \times \frac{R}{2} \sin\alpha = \left[R - \frac{R}{2}\cos\alpha\right] f$ ......(3)

$\therefore \quad \tan\theta = \frac{\sin\alpha}{2 - \cos\alpha} \Rightarrow \alpha = 60^\circ$

7. If the mass M is displaced by x from its mean position each spring further stretched by 2x.



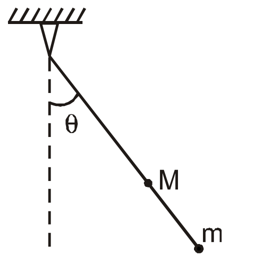
Net restoring force

$F = -8kx$

$M.a = -8kx$

$f = \frac{1}{2\pi}\sqrt{\left|\frac{a}{x}\right|} = \frac{1}{2\pi}\sqrt{\frac{8k}{M}} = \frac{1}{\pi}\sqrt{\frac{2k}{M}}$

8. Angular acceleration of rod



$\alpha = \frac{m(x+L)g\sin\theta}{m(x^2+L^2)}$

For rod to fall as fast as possible, $\frac{d\alpha}{dx} = 0$

or $x = (\sqrt{2}-1)L$

9. Let centre of disc is displaced by x from its equilibrium position(spring was in its natural length). Now calculate the torque about lowest point of disc.

$k.\frac{3}{2}R.\frac{3x}{2} = \frac{3}{2}mR^2\frac{a}{R}$

$\frac{3kx}{2m} = a$

So, $T = 2\pi\sqrt{\frac{2m}{3k}}$.

10. amplitude is obtained for v = 0

$\therefore \quad A = \sqrt{\frac{E}{a}}$

Maximum velocity is obtained for x = 0

$V_{max} = \sqrt{\frac{E}{b}} \qquad V_{max} = A\,\omega$

$$\omega = \frac{\sqrt{\frac{E}{b}}}{\sqrt{\frac{E}{a}}} = \sqrt{\frac{a}{b}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{b}{a}}$$

Alternative

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$b = \frac{m}{2}, \; a = \frac{k}{2}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{a}{b}}$

$E = \frac{1}{2}mv_{max}^2 \Rightarrow V_{max} = \sqrt{\frac{E}{b}}$

$E = \frac{1}{2}kA^2 \qquad A = \sqrt{\frac{E}{a}}$

11. $T = 2\pi\sqrt{\frac{I}{mg\ell}}, \; I = m\ell^2 + m(2\ell)^2 = 5m\ell^2$

$$= 2\pi\sqrt{\frac{5m\ell^2}{2mg\frac{3\ell}{2}}} = 2\pi\sqrt{\frac{5\ell}{3g}}$$

$\therefore \qquad L_{eq} = \frac{5\ell}{3}$

12. $Mg - f_B = F_v$

$\Rightarrow \qquad \frac{4}{3}\pi r^3(\rho_m - \rho_\ell)g = F_v$

13. **(a)** Initially

$I_1 = \frac{3}{10}mR^2 \quad \& \qquad \omega_1 = \omega$

Finally $I_2 = \frac{13}{10}mR^2 \quad \& \qquad \omega_2 = \omega_{new}$

Using conservation of Angular momentum

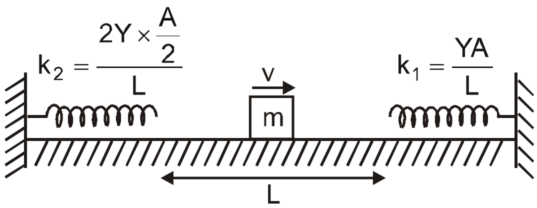
$I_1\omega_1 = I_2\omega_2$

$\omega_2 = \omega_{new} = \dfrac{3\omega}{13}$

14. Energy Density $= \dfrac{1}{2}$ stress × strain $= \dfrac{1}{2}$ Y (strain)^2 = 2880 J/m^3

15. Rod behaves as spring of spring constant $\dfrac{YA}{\ell}$

Equivalent system is:



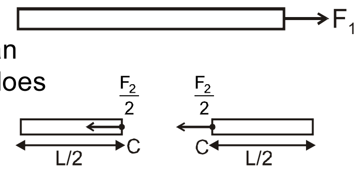
The time period of oscillations of block is

$$T = \frac{2L}{V} + \frac{1}{2}\left(2\pi\sqrt{\frac{mL}{YA}}\right) + \frac{1}{2}\left(2\pi\sqrt{\frac{mL}{2Y.A/2}}\right)$$

$$= \frac{2L}{V} + 2\pi\left(\frac{mL}{AY}\right)$$

16. The force F_1 causes extension in rod.

F_2 causes compression in left half of rod and an equal extension in right half of rod. Hence F_2 does not effectively change length of the rod.



17. Since F-r curve is continuous, so

$\left.\dfrac{dF}{dr}\right|_{p^+} = \left.\dfrac{dF}{dr}\right|_{p^-} = \left.\dfrac{dF}{dr}\right|_{p} = -\alpha$ and F(at P) = 0 so Hooke's law valid near point P.

Energy required to separate the atoms $= |\Delta U| = \left|-\int \vec{F}.d\vec{r}\right| = |\text{Area enclosed between curve and r - axis}|$

18. (A) $\therefore \quad \dfrac{dv}{dt} = -bx = v\dfrac{dv}{dx}$

$$\int_u^0 v\,dv = \int_0^x -bx\,dx$$

$$\Rightarrow \quad \left.\frac{v^2}{2}\right|_u^0 = -b\left.\frac{x^2}{2}\right|_0^x$$

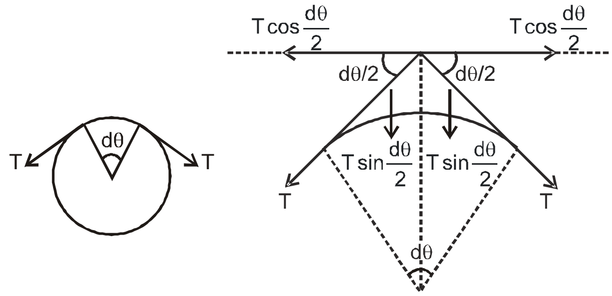
$$\Rightarrow \quad -\frac{u^2}{2} = -\frac{bx^2}{2} \quad \Rightarrow \quad x = \frac{u}{\sqrt{b}}$$

(B) F = m (– bx)

$a = -bx = -\omega^2 x$

(C) acceleration is always towards origin and acceleration is zero at origin which is the mean position of SHM.

19. Let T be the tension in the string.



$$2T\sin\frac{d\theta}{2} = \frac{m}{2\pi R}\times R.\omega^2 \cdot Rd\theta.$$

$$T = \frac{mR\omega^2}{2\pi}$$

$$Y = \frac{T/A}{\Delta\ell/\ell}$$

$$\frac{\Delta\ell}{\ell} = \frac{T}{Y.A} \Rightarrow \Delta\ell = \frac{T}{Y.A} \times \ell = \frac{m.R\omega^2}{2\pi}\times\frac{1}{Y.A}\times 2\pi R = \frac{mR^2\omega^2}{Y.A}$$

$$\frac{\Delta\ell}{\ell} = \frac{\Delta R}{R} = \frac{T}{Y.A} = \frac{m.R\omega^2}{2\pi A.Y} \Rightarrow \quad \Delta R = \frac{mR^2\omega^2}{2\pi A.Y}$$

$$V = \frac{1}{2}K.X^2 = \frac{1}{2}\left(\frac{Y.A.}{\ell}\right) \times (\Delta\ell)^2 = \frac{1}{2}\frac{Y.A}{2\pi R}\times\left(\frac{m.R^2\omega^2}{Y.A}\right)^2 = \frac{1}{4\pi}\left(\frac{m^2.R^3\omega^4}{Y.A}\right)$$

20. For disc, from torque equation

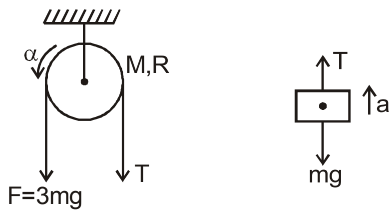
$$3\text{ mg R} - TR = \frac{mR^2}{2}\alpha \quad .... (1)$$

By application of Newton's second law on block we get,

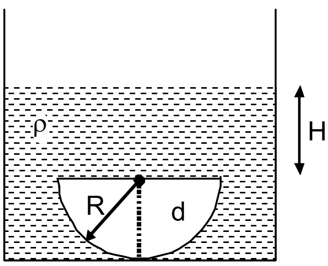
$T - mg = ma$ .... (2)

where $a = R\alpha$ ..... (3)

solving $a = \frac{4g}{3}$



21.



(a) force on flat surface depends on H

(b) Pressure at the location of curved surface depends on H

(c) Net force on hemisphere by liquid = $\left(\frac{2}{3}\pi R^3\right)(\rho)g$

22. At t = 0

Displacement x = $x_1 + x_2$ $= 4 \sin\frac{\pi}{3} = 2\sqrt{3}$ m.

Resulting Amplitude A = $\sqrt{2^2 + 4^2 + 2(2)(4)\cos\pi/3} = \sqrt{4+16+8} = \sqrt{28} = 2\sqrt{7}$ m

Maximum speed = $A\omega$ = $20\sqrt{7}$ m/s

Maximum acceleration = $A\omega^2$ = $200\sqrt{7}$ m/s²

Energy of the motion = $\frac{1}{2} m\omega^2 A^2$ = **28 J Ans.**

23. Applying conservation of the angular momentum of the system of three rods about midpoint of the rod CD .

$\Rightarrow$ m x 5 x 1 + m x 5 x 1 = $\left[2\left(\frac{m2^2}{12} + m(\sqrt{2})^2\right) + \frac{m2^2}{12}\right] \Rightarrow \omega = \frac{30}{15}$ = 2 rad/sec.

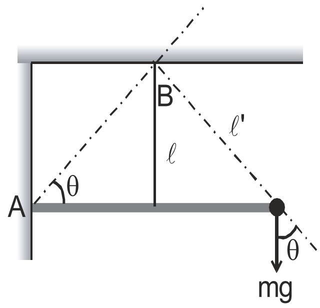
24. The bob will execute SHM about a stationary axis passing through AB. If its effective length is ℓ' then

$T = 2\pi\sqrt{\frac{\ell'}{g'}}$

$\ell' = \ell/\sin\theta = \sqrt{2}\,\ell$ (because θ = 45)

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2\ell}{g}} \quad = 2\pi\sqrt{\frac{2\times0.2}{10}} = \frac{2\pi}{5}$ s.



25. From conservation of angular momentum.

$$mu\,\frac{L}{2} + mu\,\frac{L}{2} = \left[2m\frac{L^2}{12} + m\left(\frac{L}{2}\right)^2 + \left(\frac{L}{2}\right)^2\right]\omega$$

$$muL = \left[\frac{mL^2}{6} + \frac{mL^2}{4} + \frac{mL^2}{4}\right]\omega = \frac{2mL^2}{3}\omega \quad \text{or} \quad \omega = \frac{3u}{2L} = \frac{3\times6}{2\times1} = 9 \text{ rad/s}$$

26. N = mg

f = ma

As f must be static friction (No slip condition)

$f \le \mu N \quad \Rightarrow \quad ma \le \mu\, mg$

or $ma_o \le \mu mg$

$\therefore \quad mA\omega^2 \le \mu mg$

$\therefore \quad \omega \le \sqrt{\frac{\mu g}{A}}$

$\omega = \frac{2\pi}{T} \le \sqrt{\frac{\mu g}{A}}$

$\therefore \quad T \geq 2\pi\sqrt{\frac{A}{\mu g}} \quad \Rightarrow \quad \mu \geq \frac{4\pi^2 A}{gT^2}$

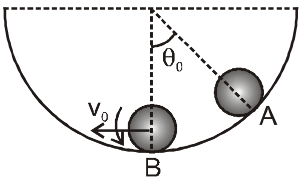
27. The x coordinates of the particles are

$x_1 = A_1 \cos \omega t$, $x_2 = A_2 \cos \omega t$

separation $= x_1 - x_2 = (A_1 - A_2) \cos \omega t = 12 \cos \omega t$

Now $x_1 - x_2 = 6 = 12 \cos \omega t$

$\Rightarrow \quad \omega t = \frac{\pi}{3} \Rightarrow \frac{2\pi}{12} . t = \frac{\pi}{3} \quad \Rightarrow \quad t = 2s$ **Ans.**

28. 

Using conservation of mechanical energy

. $E_A = E_B$

$mg\,4R\,(1-\cos\theta) = \frac{7}{10} m v_0^2 \Rightarrow 8\,mg\,R \sin^2\frac{\theta}{2} = \frac{7}{10} m v_0^2$

since θ is very small

$\frac{7}{10} v_0^2 = 2\,gR\,\theta_0^2 \qquad v_0^2 = \frac{20gR}{7}\theta_0^2$

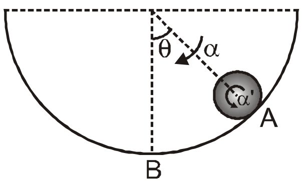
Linear amplitude of SHM $a = 4R\theta_0 \Rightarrow \theta_0 = \frac{a}{4R}$

$v_0^2 = \frac{20gR}{7}\frac{a^2}{16R^2} = \frac{5}{28}\frac{g}{R}a^2$

comparing $v_0^2 = \omega^2 a^2$

$\omega = \sqrt{\frac{5g}{28R}}, \qquad T = 2\pi\sqrt{\frac{28R}{5g}}$

Alternate solution :



$mg\sin\theta - f = m\,\alpha' R.$

$f\,R = \frac{2}{5} mR^2(\alpha')$

acceleration of center of mass of solid sphere $= \alpha' R = \alpha 4R$

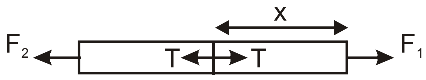
solving above 3 equations $\Rightarrow mg\sin\theta = \frac{28}{5} m\alpha R$

For small θ

$\alpha = \frac{5g\theta}{28R}$

$$\omega = \sqrt{\frac{5g}{28R}}\,, \qquad T = 2\pi\sqrt{\frac{28R}{5g}}$$

29. $dL = \frac{T}{A}\frac{dx}{y}$

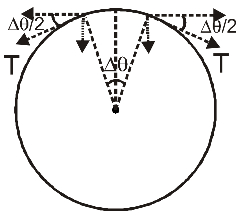


$$T = F_1 - (F_1 - F_2)\frac{x}{L}$$

$$\int_0^{\Delta L} dl = \frac{(F_1 + F_2)L}{2Ay} = 1 \times 10^{-9}\ \text{m}$$

30. $2T \sin\frac{\Delta\theta}{2} = dm \times \omega^2 r$

$$2T\left(\frac{\Delta\theta}{2}\right) = \rho \times A \times r\Delta\theta \times \omega^2 \times r$$

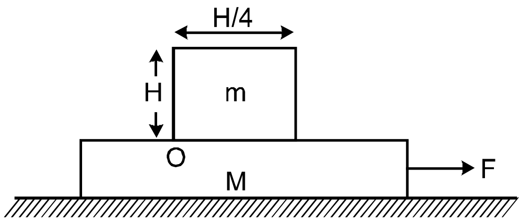


$$\sigma = \frac{T}{A} = \rho r^2 \omega^2$$

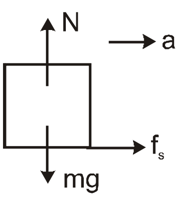
$$\therefore \qquad \omega = \frac{1}{r}\sqrt{\frac{\sigma}{\rho}} = 2\ \text{rad/s}$$

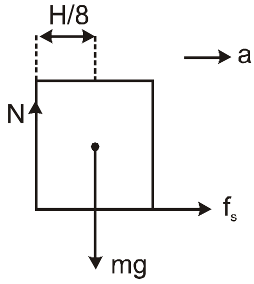
31. Let the original length of the string be L.
Applying $F = kx$, we have $4 = k(5 - L)$
$5 = k(6 - L)$
$9 = k(2X - L)$. From these equations $x = 5$

32. The block has two tendencies,



(i) to slide w.r.t. plank
(ii) to topple over the point O maximum acceleration for sliding





$f_s = ma \le f_L$
$ma \le \mu mg$

$a \le \frac{g}{3}$ $\qquad a_{max} = \frac{g}{3}$

Maximum acceleration for toppling,
$N = mg$, $\qquad f_s = ma$

$$N.\left(\frac{H}{8}\right) = \frac{H}{2}f_{s}, \qquad a_{max} = \frac{g}{4}$$

So, the block will topple before sliding. Hence, $f_{max} = (M + m)\frac{g}{4}$.

33 to 35 Time taken by particle to go from

$x = 0$ to $x = A/2$ is $\frac{T}{12}$

$\therefore \qquad$ time interval $= \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$

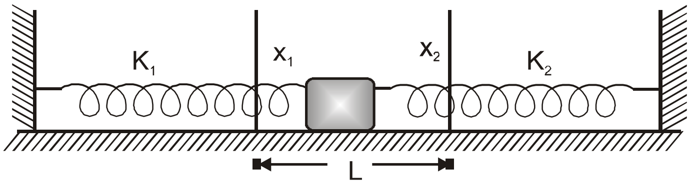
$$= \frac{7}{12}.2\pi\sqrt{\frac{m}{K_1}} = \frac{7\pi}{6}\sqrt{\frac{m}{K_1}}.$$

Assume, maximum compression in right spring is x. Hence,

$$\frac{1}{2}K_1(2L)^2 = \frac{1}{2}K_1(L + x)^2 + \frac{1}{2}K_2x^2$$

put $\quad K_2 = \frac{3}{4}K_1$, we get $\quad x = \frac{6L}{7}$.

When mass m is in equilibrium both spring will be in extended state.

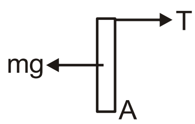


$K_1x_1 = K_2x_2 \qquad$ and $\qquad x_1 + x_2 = L$

$x_1 = \frac{3L}{7}$.

36. Torque about 'A'

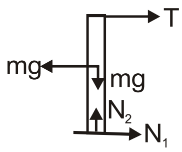
$$\frac{mg\ell}{2} - T\ell = 0$$



$T = \frac{mg}{2}$ newton

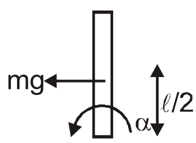
37. $N_2 = mg$
$N_1 + T = mg$

$N_1 = \frac{mg}{2}, \qquad N = \sqrt{N_1^2 + N_2^2} = mg\frac{\sqrt{5}}{2}$



38. $mg\frac{\ell}{2} = \left(\frac{m\ell^2}{3}\right)\alpha$

$\Rightarrow \qquad \alpha = \frac{3g}{2\ell}$



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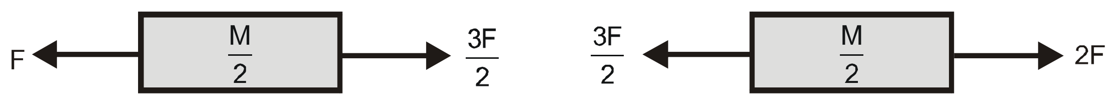
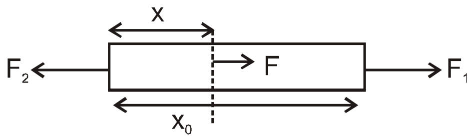


$$\Delta\ell_1 = \left(\frac{3F}{2}\right)\frac{\frac{L}{2}}{2AY} + \frac{F\frac{L}{2}}{2AY} = \frac{5FL}{8AY}$$

$$\Delta\ell_2 = \frac{2F\frac{L}{2}}{2AY} + \frac{\frac{3F}{2}\frac{L}{2}}{2AY} = \frac{7FL}{8AY}$$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{5}{7}$$

40. $F = \left(\frac{F_1 - F_2}{x_0}\right)x + F_2 = \alpha x + \beta$



Energy density at any x

$$\frac{dU}{dV} = \frac{1}{2}\left(\frac{\alpha x + \beta}{A}\right)\left(\frac{\alpha x + \beta}{AY}\right) = \frac{1}{2A^2Y}(\alpha x + \beta)^2$$

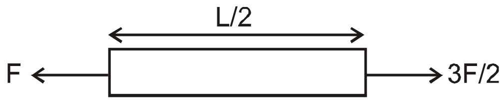
Energy stored in small segment dx

$$dU = \frac{1}{2A^2Y}(\alpha^2x^2 + \beta^2 + 2\alpha\beta x)\,Adx$$

$$U = \int dU = \frac{1}{2AY}\int_0^{x_0}(\alpha^2x^2 + \beta^2 + 2\alpha\beta x)dx = \frac{1}{2AY}\left(\frac{\alpha^2x_0^3}{3} + \beta^2x_0 + \beta x_0^2\right)$$

Consider section PQ

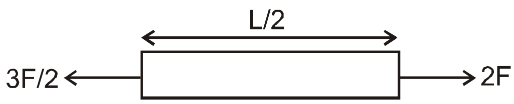
$\alpha = F/L$, $\beta = F$, $x_0 = L/2$



$$U_1 = \frac{19F^2L}{48AY}$$

Consider secton QR

$\alpha = F/L$, $\beta = 3F/2$, $x_0 = L/2$



$$U_2 = \frac{37F^2L}{48AY}$$

41. $U = \frac{19F^2L}{48AY} + \frac{37F^2L}{48AY} = \frac{7F^2L}{6AY}$

42. $F = T.4\ell$ $\quad A = 400\ cm^2$, $\ell = 20\ cm = 0.2\ m$

$$= \frac{8}{100} \times 4 \times \frac{2}{10}$$

$$= \frac{64}{100} = 0.064\ N \cong 0.06\ N.$$

(B) $\quad W = T.4\pi r^2 (n^{1/3} - 1)$

$$= \frac{8}{100} \times 4\pi \times \frac{1}{100 \times 100} \times (9)$$

$= 32 \times 9\pi \times 10^{-6}$

$= 90432 \times 10^{-6}$

= 0.09432 Joule

(C) $\quad W = 2 \times T4\pi\,[(2R)^2 - R^2]$

$= 8\pi TR^2 \times 3$

$$= 24\pi\frac{8}{100} \times \frac{1}{100 \times 100}$$

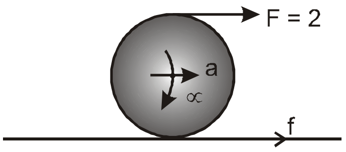
$$= \frac{59088}{1000000} = 0.059088\ \ 0.06\ \text{Joule}$$

(D) $h = \dfrac{2T.\cos\theta}{r\rho g}$

$= 2 \times \dfrac{8}{100} \times \dfrac{1\times10^4}{5\times10^3\times10}$

$= \dfrac{32}{10}\times\dfrac{10^4}{10^6}$ = 32× 10^{4-7} = 0.032 m = 3.2 cm

43. (A)



Let friction be static

F + f = ma

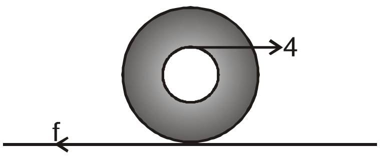
$FR - fR = \dfrac{2}{5}mR^2\alpha$

a = Rα

$f = \dfrac{6}{7}N$

f_L = 0.5N ⇒ friction is kinetic

(B)



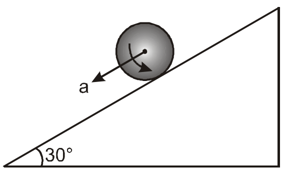
4 – f = ma

$4\times\dfrac{1}{2}+f\times1 = 2a$

f = 2N

f_L = 3N

(C)

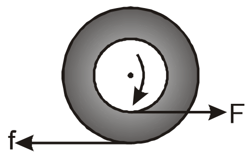


$a = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}} = \dfrac{10\times\dfrac{1}{2}}{1+\dfrac{1}{2}} = \dfrac{10}{3}$ m/s²

mg sin θ – f = ma

$f = \dfrac{mg}{2} - ma = 10 - \dfrac{20}{3} = \dfrac{10}{3}N$

$f_L = u_sN = \dfrac{2}{5}\times2\times10\times\dfrac{\sqrt{3}}{2} = 4\sqrt{3}$ N Static

(D) 

$f_L = 5N$
$f = 10/3\,N$ static
$F - f = ma \Rightarrow 4 - f = 1a$

$f \times 1 - 4\times\frac{1}{2} = 2\times\frac{a}{R} \Rightarrow f - 2 = 2a; \quad a = 2/3\ m/s^2$

44. $KE_{max} = \frac{1}{2}mv_{max}^2 = TE$

$\Rightarrow \quad v_{max} = \sqrt{\frac{2\times 2}{1}} = 2\ m/s$

amplitude $A = \frac{v_{max}}{\omega} = 2m.$

$x = A\sin\omega t = 2\sin t$

$v = 2\cos t = \sqrt{4 - x^2}$

(P) $v = \sqrt{2}$ m/s $\Rightarrow x = \pm\sqrt{2}$ m.

(Q) $KE = \frac{1}{2}mv^2 \Rightarrow 1 = \frac{1}{2}\times 1\times v^2 \Rightarrow v = \sqrt{2}$ m/s.

$\therefore x = \pm\sqrt{2}$ m.

(R) at $t = \pi/6$ s, $x = 2\sin\pi/6 = 1m.$

(S) $KE = \frac{3}{2} \Rightarrow 1.5 = \frac{1}{2}\times mv^2$

$\Rightarrow \quad v = \sqrt{3} \Rightarrow x = \pm 1m.$

45. (P) $T = 2\pi\sqrt{\frac{m}{k}}$ $m\uparrow \quad T\uparrow$

$E = \frac{1}{2}kA^2$

(Q) $E = \frac{1}{2}kA^2$ $A\uparrow \quad E\uparrow$

(R) $T = 2\pi\sqrt{\frac{m}{k}}$ $k\uparrow \quad T\downarrow$

$E = \frac{1}{2}kA^2$ $k\uparrow \quad E\uparrow$

(S) $T = 2\pi\sqrt{\frac{m}{k_{eq}}}$ $k_{eq}\uparrow \quad T\downarrow$

$k_{eq} = 2k$

$E = \frac{1}{2}k_{eq}A^2$ $k_{eq}\uparrow \quad E\uparrow$